

例 27 证明不等式

$$f(x) = \frac{1}{2}x^2 - \frac{m}{2}\ln(1+2x) + m, \quad m < 0$$

证明 $f(x)$ 在 $(-\frac{1}{2}, +\infty)$ 上恒有 $f(x) > e+1$

$$m = -\frac{e}{2} \quad x \in (-\frac{1}{2}, \frac{e-1}{2}] \quad f(x_0) > e+1 \quad m < -\frac{e}{2}$$

$$m = -1 \quad x_1, x_2 \in (0, 1) \quad x_1 \neq x_2 \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < \frac{1}{3}$$

$$f(x) = \frac{1}{2}x^2 - \frac{m}{2}\ln(1+2x) + m$$

$$\therefore f(x) = x - \frac{m}{1+2x} + m = \frac{2x^2 + (2m+1)x}{1+2x} = \frac{x(2x+2m+1)}{2x+1}$$

$$\textcircled{1} \quad 2m+1=0 \quad m = -\frac{1}{2} \quad f(x) \geq 0$$

$$f(x) \geq (-\frac{1}{2}, +\infty)$$

$$\textcircled{2} \quad 0 < 2m+1 < 1 \quad -\frac{1}{2} < m < 0$$

$$f(x) \geq (-\frac{1}{2}, -\frac{2m+1}{2}) \cup (0, +\infty)$$

$$(-\frac{2m+1}{2}, 0)$$

$$\textcircled{3} \quad m < -\frac{1}{2}$$

$$f(x) \geq (-\frac{1}{2}, 0) \cup (-\frac{2m+1}{2}, +\infty)$$

$$(0, -\frac{2m+1}{2})$$

$$m = -\frac{e}{2}$$

$$\therefore \frac{e-1}{2} < -\frac{2m+1}{2}$$

$$\forall x \in (-\frac{1}{2}, \frac{e+1}{2}] \quad f(x) > e+1$$

$$f(0) > e+1$$

$$-2m > e+1$$

$$m < -\frac{e+1}{2}$$

$$m = -1$$

$$f(x) = x + \frac{1}{2x+1} - 1 \quad (0, \frac{\sqrt{2}-1}{2})$$

$$(\frac{\sqrt{2}-1}{2}, 1)$$

$$f(0) = 0 \quad f'(1) = \frac{1}{3}$$

$$f(x) < \frac{1}{3} \quad x \in (0, 1)$$

$$f(x) < \frac{1}{3}$$

$$\forall x_1, x_2 \in (0, 1) \quad x_1 \neq x_2 \quad \frac{f(x_1) - f(x_2)}{x_1 - x_2} < \frac{1}{3}$$

$$f(x) = \frac{a - \ln x}{x} \quad (1, f(1))$$

$$a > f(x)$$

$$\forall x_1, x_2 \in [e^2, +\infty) \quad \left| \frac{f(x_1) - f(x_2)}{x_1 - x_2} \right| > \frac{k}{x_1 x_2}$$

$$f(x) = \frac{a - \ln x}{x}$$

$$\therefore f(x) = \frac{x(-\frac{1}{x}) - (a - \ln x)}{x^2} = \frac{-1 - a + \ln x}{x^2}$$

$$f'(1) = 0$$

$$\therefore 1 - a + \ln 1 = 0$$

$$a = -1$$

$$f'(x) = 0 \quad \ln x = 0$$

$$x = 1$$

$$f(x) \text{ 在 } x=1 \text{ 处取得极大值 } f(1) = -1$$

$$\left| \frac{f(x_1) - f(x_2)}{x_1 - x_2} \right| > \frac{k}{x_1 - x_2} \quad \left| \frac{\frac{1}{x_1} - \frac{1}{x_2}}{\frac{1}{x_1} - \frac{1}{x_2}} \right| > k$$

$$g\left(\frac{1}{x}\right) = f(x) \quad g(x) = x - x \ln x \quad x \in (0, e^2]$$

$$g'(x) = -\ln x \quad x \in (0, e^2] \quad g'(x) = -\ln x \cdot 2$$

$$\left| \frac{f(x_1) - f(x_2)}{\frac{1}{x_1} - \frac{1}{x_2}} \right| > 2$$

$$k \text{ 在 } (-\infty, 2] \text{ 上恒成立}$$

$$f(x) = \frac{1 + \ln x}{x}$$

$$\left(t + \frac{2}{3}\right) \quad t > 0 \quad f(x) \text{ 在 } t \text{ 处取得极大值}$$

$$x_1, x_2 \in [e, +\infty) \quad \left| \frac{f(x_1) - f(x_2)}{x_1 - x_2} \right| \leq k \left| \frac{1}{x_1} - \frac{1}{x_2} \right| \quad k \text{ 在 } [e, +\infty) \text{ 上恒成立}$$

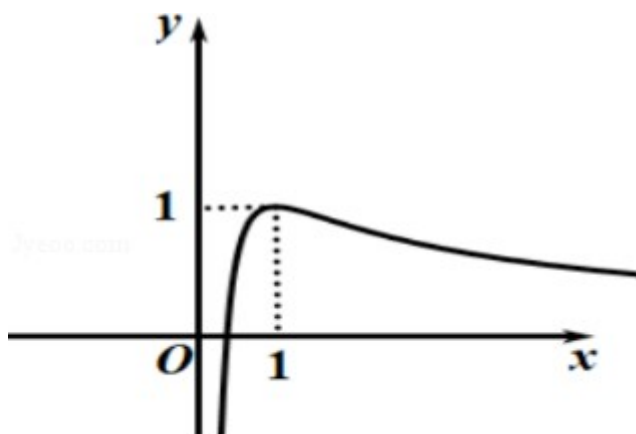
$$f(x) \text{ 在 } (0, +\infty) \text{ 上取得极大值 } f(x) = -\frac{\ln x}{x^2}$$

$$f'(x) > 0 \Rightarrow 0 < x < 1 \quad f'(x) < 0 \Rightarrow x > 1$$

$$f(x) \text{ 在 } (0, 1) \text{ 上取得极大值 } f(1) = 1$$

$$x \rightarrow 0 \quad y \rightarrow -\infty \quad x > 1 \quad f(x) = \frac{1 + \ln x}{x} > 0$$

$$f\left(\frac{1}{e}\right) = 0 \quad f(x) \text{ 在 } (0, 1) \text{ 上取得极大值 } f(x) \text{ 在 } (1, +\infty) \text{ 上取得极小值}$$



\therefore $(t + \frac{2}{3}), t > 0$ $f(x)$

$$\therefore \begin{cases} 1 < t + \frac{2}{3} \\ f(t) = \frac{1 + \ln t}{t} < 0 \end{cases} \quad \frac{1}{3} < t < \frac{1}{e}$$

$$t \in (\frac{1}{3}, \frac{1}{e})$$

2 1 $f(x)$ $[e^2, +\infty)$

$$\text{由 } x_1 > x_2 \dots e^2 \quad |f(x_1) - f(x_2)| \leq k \left| \frac{1}{x_1} - \frac{1}{x_2} \right| \quad f(x_2) - f(x_1) \leq k \left(\frac{1}{x_2} - \frac{1}{x_1} \right)$$

$$\therefore f(x_2) - \frac{k}{x_2} \leq f(x_1) - \frac{k}{x_1}$$

$$F(x) = f(x) - \frac{k}{x} = \frac{1 + \ln x}{x} - \frac{k}{x}$$

$$F(x) = f(x) - \frac{k}{x} \quad [e^2, +\infty)$$

$$F(x) = \frac{k - \ln x}{x} \geq 0 \quad [e^2, +\infty)$$

$$k, \ln x \quad [e^2, +\infty)$$

$$\text{由 } x \in [e^2, +\infty) \quad \ln x \quad \ln e^2 = 2$$

$$\therefore k, 2$$

$$K \in (-\infty, 2]$$

$$4 \text{ Let } f(x) = e^x, g(x) = -x^2 + 2x, \text{ and } f(x)(a \in R) \text{ on } x_1, x_2 \text{ such that } x_1 \neq x_2$$

$$1 \text{ Let } f(x) \text{ on } x=0$$

$$2 \text{ Let } g(x) \text{ on } R \text{ and } a$$

$$3 \text{ Let } f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$$

$$\text{Let } f(x) = e^x \text{ on } [1, \infty)$$

$$\text{Let } f(0) = 1 \text{ on } (0, 1)$$

$$\text{Let } f(x) \text{ on } x=0 \text{ and } y=x+1$$

$$2 \text{ Let } g(x) = -x^2 + 2x, \text{ and } g'(x) = -2x + 2$$

$$\text{Let } f(x) = e^x$$

$$\text{Let } g'(x) = -2x + 2, \text{ and } g(x) = -x^2 + 2x$$

$$\text{Let } a_n = \frac{-2x+2}{e^x}$$

$$\text{Let } h(x) = \frac{-2x+2}{e^x}$$

$$\text{Let } h(x) = \frac{2(x-2)}{e^x} = 0 \text{ on } x=2$$

$$\text{Let } x \in (-\infty, 2) \text{ and } h(x) < 0$$

$$\text{Let } x \in (2, +\infty) \text{ and } h(x) > 0$$

$$\text{Let } x=2 \text{ and } h(x)_{\min} = h(2) = \frac{-2}{e^2}$$

$a \in (-\infty, -\frac{2}{e^3}]$ 9

$f(\frac{x_1+x_2}{2}) < \frac{f(x_1)-f(x_2)}{x_1-x_2} < \frac{e^{\frac{x_1+x_2}{2}}-e^{x_2}}{x_1-x_2}$ 3

$e^{\frac{x_1+x_2}{2}} < \frac{e^{x_1+x_2}-1}{x_1-x_2} \quad x_1 > x_2 \quad t = \frac{x_1-x_2}{2}$

$e^t < \frac{e^{2t}-1}{2t} \quad (t > 0)$

$2te^t < e^{2t}-1 \quad t > 0$ 11

$h(t) = e^{2t} - 2te^t - 1$

$h'(t) = e^{2t}(2t) - 2te^t - 2e^t = 2e^{2t} - 2te^t - 2e^t = 2e^t(e^t - t - 1)$

$\varphi'(t) = e^t - t - 1 \quad t > 0 \quad \varphi'(t) = e^t - 1 > 0$

$\varphi'(t) > \varphi(0) = 0 \quad h(t) > 0$ 13

$h(t) > 0 \quad h(t) > h(0) = 0$

$2te^t < e^{2t}-1 \quad t > 0$

$e^{\frac{x_1+x_2}{2}} < \frac{e^{x_1+x_2}-1}{x_1-x_2} < \frac{f(x_1)-f(x_2)}{x_1-x_2}$ 16

$f(x) = \ln x$ 5

$g(x) = af(x) - \frac{1}{x}$ 1

$x > 0 \quad f(x), \quad ax, \quad e^x \quad a$ 2

$x_1 > x_2 > 0 \quad \frac{f(x_1)-f(x_2)}{x_1-x_2} > \frac{2x_2}{x_1^2+x_2^2}$ 3

$$\text{□□□□□□□□1□} \quad f(x) = \ln x \quad \therefore \quad g(x) = a \ln x - \frac{1}{x} \quad \square$$

$$\square \quad g'(x) = \frac{a}{x} + \frac{1}{x^2} = \frac{ax+1}{x^2} \dots \quad \square 2 \square \square$$

$$\square \quad x > 0 \quad \square \square \square \quad a > 0 \quad \square \square \quad g'(x) > 0 \quad \square \square \quad g(x) \quad \square \quad (0, +\infty) \quad \square \square \square \square \square \square$$

$$\square \quad a < 0 \quad \square \square \square \quad x \in (0, -\frac{1}{a}), g'(x) > 0 \quad \square \square \square \quad g(x) \quad \square \square \square \square \square$$

$$\square \quad x \in (-\frac{1}{a}, +\infty), g'(x) < 0 \quad \square \square \square \quad g(x) \quad \square \square \square \square \square \quad \dots \quad \square 4 \square \square$$

$$\square 2 \square \square \quad \square \square \square \quad x > 0 \quad \square \square \square \square \square \square \square \square \quad x > 0 \quad \square \square \square \square \quad f(x), \quad ax, \quad e^x \quad \square \square \square \square$$

$$\therefore \quad \frac{\ln x}{x}, \quad a, \quad \frac{e^x}{x} \quad \square \quad x > 0 \quad \square \square \square \square \square \square \square \square \square \square \quad \left(\frac{\ln x}{x}\right)_{\min}, \quad a, \quad \left(\frac{e^x}{x}\right)_{\min} \quad \square \quad \dots \quad \square 5 \square \square$$

$$\square \quad h(x) = \frac{\ln x}{x}, h'(x) = \frac{1 - \ln x}{x^2} \quad \square \square \quad x \in (0, e) \quad \square \square \quad h(x) > 0 \quad \square$$

$$\square \quad x \in (e, +\infty) \quad \square \square \quad h(x) < 0 \quad \square \therefore \square \quad x = e \quad \square \square \quad h_{\min}(x) = \frac{1}{e} \quad \square \quad \dots \quad \square 7 \square \square$$

$$\square \quad t(x) = \frac{e^x}{x}, t'(x) = \frac{xe^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2} \quad \square \square \quad x \in (0, 1) \quad \square \square \quad t(x) < 0 \quad \square$$

$$\square \quad x \in (1, +\infty) \quad \square \square \quad t'(x) > 0 \quad \square \square \square \quad x = 1 \quad \square \square \quad t_{\min}(x) = e \quad \square \quad \dots \quad \square 9 \square \square$$

$$\square \quad \frac{1}{e}, \quad a, \quad e \quad \square \square \square \square \square \quad a \quad \square \square \square \square \square \square \quad \left[\frac{1}{e}, e\right] \dots \quad \square 10 \square \square$$

$$\square 3 \square \square \quad x_1 > x_2 > 0 \quad \square \square \quad \frac{f(x_1) - f(x_2)}{x_1 - x_2} > \frac{2x_2}{x_1^2 + x_2^2} \quad \square \square \square \quad \ln \frac{x_1}{x_2} > \frac{2\left(\frac{x_1}{x_2} - 2\right)}{\left(\frac{x_1}{x_2}\right)^2 + 1} \quad \square \quad \dots \quad \square 11 \square \square$$

$$\square \quad t = \frac{x}{x_2} > 1 \quad \square \square \quad u(t) = \ln t - \frac{2t-2}{t^2+1} \quad \square \square \quad u(t) = \frac{(t-1)(t+2t-1)}{t(t^2+1)^2} \quad \square$$

$$\square \quad t \in (1, +\infty) \quad \square \square \quad t-1 > 0 \quad \square \quad t+2t-1 > 0 \quad \square \therefore u(t) > 0 \dots \quad \square 13 \square \square$$

$$\therefore u(t) \quad \square \quad (1, +\infty) \quad \square \square \square \square \square \square \quad \therefore u(t) > u \quad \square 1 \square = 0 \quad \square$$

$$\therefore \frac{f(x_1) - f(x_2)}{x_1 - x_2} > \frac{2x_2}{x_1^2 + x_2^2} \quad \dots \quad \boxed{14} \quad \square$$

6. $f(x) = \frac{a - 2\ln x}{x^2}$ $(1 \leq f \leq 1)$ $y = -4x + 1$

1. a $f(x)$

2. $x_1, x_2 \in (0, \frac{1}{e}]$ $|\frac{f(x_1) - f(x_2)}{x_1^2 - x_2^2}| > \frac{k}{x_1^2 + x_2^2}$ k

1. $f(x) = \frac{-2 - 2a + 4\ln x}{x^2}$ $(x > 0)$

$(1 \leq f \leq 1)$ $y = -4x + 1$

$f(1) = -4$ $\frac{-2 - 2a}{1} = -4$ $a = 1$

$f(x) = \frac{-2 - 2a + 4\ln x}{x^2} = \frac{-4 + 4\ln x}{x^2} = 0$

$x = e$

$f(x) > 0$ $x > e$

$f(x)$ $(e, +\infty)$

$f(x) < 0$ $0 < x < e$

$f(x)$ $(0, e)$

$f(x)$ $x = e$ $f(e) = -\frac{1}{e^2}$ $\boxed{6}$

$$\left| \frac{f(x_1) - f(x_2)}{x_1^2 - x_2^2} \right| > \frac{k}{x_1^2 x_2^2} \quad \left| \frac{f(x_1) - f(x_2)}{\frac{1}{x_1^2} - \frac{1}{x_2^2}} \right| > k$$

$$g\left(\frac{1}{x}\right) = f(x) \quad g(x) = x + x \ln x \quad x \in [e^2, +\infty) \quad g'(x) = 2 + \ln x$$

$$x \in [e^2, +\infty) \quad g'(x) = 2 + \ln x. 4$$

$$\left| \frac{f(x_1) - f(x_2)}{\frac{1}{x_1^2} - \frac{1}{x_2^2}} \right| > 4$$

$$\therefore k \in (-\infty, 4] \quad 12$$

$$7 \quad f(x) = e^{kx} - 2x \quad k$$

$$k=1 \quad f(x)$$

$$f(x) \cdot 1 \quad k$$

$$f(x) \quad x_1 < x_2 < x_3 \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$(f) \quad f(x) = e^x - 2x$$

$$\therefore f(x) = e^x - 2$$

$$f(x) = 0 \quad x = \ln x$$

$$\therefore x < \ln 2 \quad f(x) < 0 \quad f(x) \quad (-\infty, \ln 2) \quad x > \ln 2 \quad f(x) > 0 \quad f(x) \quad (\ln 2, +\infty)$$

$$\therefore f(x) \quad f(\ln 2) = 2 - 2\ln 2$$

$$(II) \quad f(x) = ke^{kx} - 2$$

$$\textcircled{1} \quad k < 0 \quad f(x) \text{ 在 } R \text{ 上单调递增}$$

$$\square \quad x > 0 \quad f(x) < f(0) = 1$$

$$\therefore f(x) < 1$$

$$\textcircled{2} \quad k > 0 \quad f(x) = 0 \quad x = \frac{1}{k} \ln \frac{2}{k}$$

$$\square \quad x < \frac{1}{k} \ln \frac{2}{k} \quad f(x) < 0 \quad f(x) \text{ 在 } (-\infty, \frac{1}{k} \ln \frac{2}{k}) \text{ 上单调递减} \quad x > \frac{1}{k} \ln \frac{2}{k} \quad f(x) > 0 \quad f(x) \text{ 在 } (\frac{1}{k} \ln \frac{2}{k}, +\infty) \text{ 上单调递增}$$

$$\therefore f(x) \text{ 在 } x = \frac{1}{k} \ln \frac{2}{k} \text{ 处取得极小值}$$

$$\square \quad f(x) > 1 \quad f(x) \text{ 在 } (-\infty, +\infty) \text{ 上恒成立}$$

$$\therefore \frac{2}{k} - \frac{2}{k} \ln \frac{2}{k} > 1$$

$$\square \quad g(x) = x - \ln x \quad x > 0 \quad g(x) > 1$$

$$\square \quad g(x) = 1 - \ln x - 1 = -\ln x$$

$$\therefore g(x) \text{ 在 } (0, 1) \text{ 上单调递减, 在 } (1, +\infty) \text{ 上单调递增}$$

$$\therefore g(x) \text{ 在 } x = 1 \text{ 处取得极小值}$$

$$\therefore \frac{2}{k} = 1$$

$$\therefore k = 2$$

$$(III) \quad 1:$$

$$\square \quad f(x_2) = ke^{kx_2} - 2 > 0 \quad k > 0$$

$$\square \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < f(x_2)$$

$$\square \quad x_2 - x_1 > 0 \quad \square$$

$$\square \square \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2) \quad \square$$

$$\square \square \square \quad f(x_2) - f(x_1) < (x_2 - x_1)(ke^{kx_2} - 2) \quad \square \square \square \quad e^{kx_2} - e^{kx_1} < k(x_2 - x_1)e^{kx_2} \quad \square$$

$$\square \square \square \quad 1 - e^{k(x_1 - x_2)} < k(x_2 - x_1) \quad \square \square \square \quad e^{k(x_2 - x_1)} - k(x_2 - x_1) - 1 > 0 \quad \square$$

$$\square \quad h(x) = e^x - x - 1 \quad \square$$

$$\square \quad h(x) = e^x - 1 < 0 \quad \square$$

$$\therefore h(x) \quad \square \quad (-\infty, 0) \quad \square \square \square \square \square \square$$

$$\therefore h(x) > h(0) = 0 \quad \square$$

$$\square \quad x = k(x_1 - x_2) < 0 \quad \square$$

$$\therefore h(k(x_1 - x_2)) > 0 \quad \square$$

$$\therefore \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2) \quad \square$$

$$\square \square \square \square \quad f(x_2) < \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \square$$

$$\therefore \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < \frac{f(x_2) - f(x_2)}{x_2 - x_2} \quad \square$$

$$(III) \quad \square \square \quad 2:$$

$$\square \quad f(x_0) = ke^{kx_0} - 2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \square \square \quad x_0 = \frac{1}{k} \ln \left(\frac{2}{k} + \frac{f(x_2) - f(x_1)}{k(x_2 - x_1)} \right) \quad \square$$

$$\square \square \square \square \quad x_1 < x_0 < x_2 \quad \square$$

$$\square \quad g(x) = f'(x) = ke^{kx} - 2 \quad \square \quad g(x) = k^2 e^{kx} > 0 \quad \square \square \square \square \quad f(x) \quad \square \square \square \square \square$$

$$f(x_2) - \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1}{x_2 - x_1} [(x_2 - x_1) f'(x_2) - (f(x_2) - f(x_1))]$$

$$\square \square \square \square \quad k(x) = (x_2 - x_1) f'(x_2) - (f(x_2) - f(x_1)) \quad \square \quad (x, x_2) \quad \square$$

$$k(x) = f'(x) - f'(x_2), \quad 0 \quad \square$$

$$k(x_2) = 0 \quad \square \square \quad x, x_2 \quad \square \square \quad k(x) > 0 \quad \square \square \square \quad f(x_2) - \frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0 \quad \square$$

$$\square \square \square \square \quad f(x_1) - \frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0 \quad \square$$

$$\square \quad f(x_1) < f(x_0) < f(x_2) \quad \square$$

$$\square \quad f(x) \quad \square \square \square \square \square \square \quad x_1 < x_0 < x_2 \quad \square \square \square \square \square \quad (x_1 \quad x_2) \quad \square \square \square \quad x_0 \quad \square \quad f(x_0) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \square$$

$$\square \square \square \square \square \square \quad (x_2 \quad x_0) \quad \square \square \square \quad x' \quad \square \quad f(x') = \frac{f(x_0) - f(x_2)}{x_0 - x_2} \quad \square$$

$$\square \quad f(x) \quad \square \square \square \square \square \square \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < \frac{f(x_0) - f(x_2)}{x_0 - x_2} \quad \square$$

$$8 \square \square \square \square \square \quad f(x) = e^{kx} - 2 \quad (k \in \mathbb{R} \quad k \neq 0) \quad \square$$

$$\square 1 \square \square \square \square \square \square \quad x \in \mathbb{R} \quad \square \square \square \quad f(x) \dots 1 \quad \square \square \quad k \square \square \square$$

$$\square 2 \square \square \square \square \square \quad f(x) \quad \square \square \square \square \square \square \square \square \square \square \square \square \quad x_1 \quad x_2 \quad x_3 \quad (x_1 < x_2 < x_3) \quad \square \square \square \square \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2) < \frac{f(x_3) - f(x_2)}{x_3 - x_2} \quad \square$$

$$\square \square \square \square \square \square \square 1 \quad \square \quad f(x) \quad \square \square \square \square \square \quad \mathbb{R} \quad \square \quad f(x) = ke^{kx} - 2 \quad \square$$

$$\textcircled{1} \quad \square \quad k < 0 \quad \square \square \quad f(x) \quad \square \square \square \square \square \square \quad f(x) \quad \square \quad \mathbb{R} \quad \square \square \square \square \square \square \square$$

$$\square \quad x > 0 \quad \square \quad f(x) < f(0) = 1 \quad \square$$

$$\therefore \square \square \square \quad f(x) \dots 1 \quad \square \square \square \square$$

$$\textcircled{2} \quad \square \quad k > 0 \quad \square \square \square \quad f(x) = 0 \quad \square \square \quad x = \frac{1}{k} \ln \frac{2}{k} \quad \square$$

$$\square \quad x < \frac{1}{k} \ln \frac{2}{k} \quad \square \square \quad f(x) < 0 \quad \square \square \quad f(x) \quad \square \quad (-\infty, \frac{1}{k} \ln \frac{2}{k}) \quad \square \square \square \square \square$$

$$\square \quad x > \frac{1}{k} \ln \frac{2}{k} \quad \square \square \quad f(x) > 0 \quad \square \square \quad f(x) \quad \square \quad (\frac{1}{k} \ln \frac{2}{k}, +\infty) \quad \square \square \square \square \square$$

$$\therefore f(x)_{\min} = f(\frac{1}{k} \ln \frac{2}{k}) = \frac{2}{k} - \frac{2}{k} \ln \frac{2}{k} \quad \square$$

$$\square \quad f(x) \dots 1 \quad \square \square \square \square \square \quad f(x)_{\min} \dots 1 \quad \square$$

$$\therefore \frac{2}{k} - \frac{2}{k} \ln \frac{2}{k} \dots 1 \quad \square$$

$$\square \square \square \square \quad g(x) = x - x \ln x \quad (x > 0) \quad \square$$

$$\therefore g'(x) = 1 - \ln x - 1 = -\ln x \quad \square$$

$$\therefore g(x) \quad \square \quad (0, 1) \quad \square \square \square \square \square \square \square \quad (1, +\infty) \quad \square \square \square \square \square \square \square$$

$$\therefore g(x), g' \quad \square \quad 1 \quad \square = 1 \quad \square \square \square \square \square \quad x = 1 \quad \square \square \square \square \square \square \square \quad 1 \quad \square$$

$$\therefore \frac{2}{k} = 1 \quad \square$$

$$\therefore k = 2 \quad \square$$

$$\square \square \square \square \square \square \square \square \quad f(x_2) = ke^{kx_2} - 2, 0 \quad \square \square \quad k > 0 \quad \square$$

$$\square \square \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2) \quad \square$$

$$\square \quad x_2 - x_1 > 0 \quad \square$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2)$$

$$f(x_2) - f(x_1) < (x_2 - x_1)(ke^{kx_2} - 2) \quad ke^{kx_2} - e^{kx_1} < k(x_2 - x_1)ke^{kx_2}$$

$$1 - e^{k(x_1 - x_2)} < k(x_2 - x_1) \quad e^{k(x_2 - x_1)} - k(x_2 - x_1) - 1 > 0$$

$$h(x) = e^x - x - 1$$

$$h(x) = e^x - 1 > 0$$

$$\therefore h(x) > 0 \quad (-\infty, 0)$$

$$\therefore h(x) > h(0) = 0$$

$$x = k(x_1 - x_2) < 0$$

$$\therefore h(k(x_1 - x_2)) > 0$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2)$$

$$f(x_2) < \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} < f'(x_2) < \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

$$f(x) = e^{ax} - x - 1 \quad f(x) \geq 0$$

$$a \geq 0$$

$$f(x) \geq 0 \quad \forall x \in \mathbb{R} \quad A(x_1, f(x_1)) \in AB \quad k \quad x_2 \in (x_1, x_2)$$

$$f(x_0) = k \quad x_0 \quad x_1 \quad x_2$$

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{L'Hôpital's Rule} \quad \lim_{x \rightarrow 0^+} \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1$$

$$a > 0 \quad f(x) = ae^{ax} - 1 \quad f(x) = ae^{ax} - 1 = 0 \quad x = \frac{-\ln a}{a}$$

$$\square \quad x < \frac{-\ln a}{a} \quad \square \square \quad f(x) < 0 \quad \square \square \square \quad f(x) \quad \square \square \square \square \square \square \quad x > \frac{-\ln a}{a} \quad \square \square \quad f(x) > 0 \quad \square \square \square \quad f(x) \quad \square \square \square \square \square \square$$

$$X = -\frac{\ln a}{a} \quad f\left(-\frac{\ln a}{a}\right) = \frac{1}{a} + \frac{\ln a}{a} - 1$$

$$\frac{1}{a} + \frac{\ln a}{a} = 1.0 \quad (1)$$

$$g(t) = t - \ln t - 1 \quad g'(t) = -\ln t$$

$$\boxed{} \quad 0 < t < 1 \quad \boxed{} \quad g'(t) > 0 \quad \boxed{} \quad g(t) \quad \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \quad t > 1 \quad \boxed{} \quad g'(t) < 0 \quad \boxed{} \quad g(t) \quad \boxed{} \boxed{} \boxed{} \boxed{}$$

$$t=1 \quad g(a) \quad g(1)=0 \quad \frac{1}{a}=1 \quad a=1 \quad \textcircled{1}$$

$$\square\square a=1\square$$

$$(II) \quad k = \frac{f(X_2) - f(X_1)}{X_2 - X_1} = \frac{e^{y_2} - e^{y_1}}{x_2 - x_1} \cdot 1$$

$$f(x) = f(x) - k = e^x - \frac{e^x - e^x}{x_2 - x_1} y = f(x) [x_1 \quad x_2]$$

$$U(X_1) = -\frac{e^{y_1}}{X_2 - X_1} [e^{y_2 - y_1} - (X_2 - X_1) - 1] \quad U(X_2) = -\frac{e^{y_2}}{X_2 - X_1} [e^{y_1 - y_2} - (X_1 - X_2) - 1]$$

(I) $f(x) = e^x - x - 1.0$ $x=0$

$$\therefore e^{N_2 - N_1} = (X_2 - X_1) - 1.0 \quad e^{N_1 - N_2} = (X_1 - X_2) - 1.0$$

$$\therefore \mathcal{U}(X_1) < 0 \quad \mathcal{U}(X_2) > 0$$

$$x_0 \in (x_1, x_2) \quad d(x_0) = 0 \quad x_0 = \ln \frac{e^{x_2} - e^{x_1}}{x_2 - x_1}$$

$$\forall x_0 \in (x_1, x_2) \quad f(x_0) = k$$

$$10 \quad f(x) = e^{ax} - x \quad a \neq 0$$

$$1 \quad x \in R \quad f(x) \leq 1 \quad a$$

$$2 \quad f(x) \quad A(x_1, f(x_1)) \quad B(x_2, f(x_2)) \quad (x_1 < x_2) \quad AB \quad K \quad x_0 \in (x_1, x_2)$$

$$f(x_0) > k \quad x_0$$

$$1 \quad a < 0 \quad x > 0 \quad f(x) = e^{ax} - x < 1$$

$$a \neq 0 \quad \therefore a > 0$$

$$f(x) = ae^{ax} - 1 \quad f(x) = 0 \quad x = \frac{1}{a} \ln \frac{1}{a}$$

$$f(x) < 0 \quad x < \frac{1}{a} \ln \frac{1}{a} \quad f(x) > 0 \quad x > \frac{1}{a} \ln \frac{1}{a}$$

$$\therefore x = \frac{1}{a} \ln \frac{1}{a} \quad f(x) \quad f\left(\frac{1}{a} \ln \frac{1}{a}\right) = \frac{1}{a} - \frac{1}{a} \ln \frac{1}{a}$$

$$\therefore x \in R \quad f(x) \leq \frac{1}{a} - \frac{1}{a} \ln \frac{1}{a} \quad \textcircled{1}$$

$$g(t) = t - \ln t \quad g'(t) = 1 - \frac{1}{t}$$

$$0 < t < 1 \quad g'(t) < 0 \quad g(t) \quad t > 1 \quad g'(t) > 0 \quad g(t)$$

$$\therefore t = 1 \quad g(t) \quad g(1) = 1$$

$$\therefore \frac{1}{a} = 1 \quad a = 1 \quad \textcircled{2}$$

$$a \quad \{1\}$$

$$2 \quad k = \frac{e^{ax_2} - e^{ax_1}}{x_2 - x_1} - 1$$

$$\varphi(x) = f(x) - k = ae^{ax} - \frac{e^{ax_2} - e^{ax_1}}{x_2 - x_1} \quad \varphi(x) = - \frac{e^{ax_1}}{x_2 - x_1} [e^{a(x_2 - x_1)} - a(x_2 - x_1) - 1]$$

$$\varphi(x_2) = \frac{e^{ax_2}}{x_2 - x_1} [e^{a(x_1 - x_2)} - a(x_1 - x_2) - 1]$$

$$F(t) = e^{-t} - 1 \quad F(t) = e^{-t} - 1$$

$$t < 0 \quad F(t) < 0 \quad t > 0 \quad F(t) > 0$$

$$\therefore t \neq 0 \quad F(t) > F(0) = 0 \quad e^{-t} - 1 > 0$$

$$\therefore e^{a(x_2 - x_1)} - a(x_2 - x_1) - 1 > 0 \quad e^{a(x_1 - x_2)} - a(x_1 - x_2) - 1 > 0$$

$$\frac{e^{ax_1}}{x_2 - x_1} > 0 \quad \frac{e^{ax_2}}{x_2 - x_1} > 0$$

$$\therefore \varphi(x_1) < 0 \quad \varphi(x_2) > 0$$

$$\therefore c \in (x_1, x_2) \quad \varphi(c) = 0$$

$$\varphi(x) = c \quad c = \frac{1}{a} \ln \frac{e^{ax_2} - e^{ax_1}}{a(x_2 - x_1)}$$

$$x \in \left(\frac{1}{a} \ln \frac{e^{ax_2} - e^{ax_1}}{a(x_2 - x_1)}, x_2 \right) \quad f(x) > k$$

$$x_0 \in (x_1, x_2) \quad f(x_0) > k \quad x_0 \in \left(\frac{1}{a} \ln \frac{e^{ax_2} - e^{ax_1}}{a(x_2 - x_1)}, x_2 \right)$$

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